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Tropical year

A **tropical year** (also known as a **solar year**) is the time that the Sun takes to return to the same position in the cycle of seasons, as seen from Earth; for example, the time from vernal equinox to vernal equinox, or from summer solstice to summer solstice. Because of the precession of the equinoxes, the seasonal cycle does not remain exactly synchronized with the position of the Earth in its orbit around the Sun. As a consequence, the tropical year is about 20 minutes shorter than the time it takes Earth to complete one full orbit around the Sun as measured with respect to the fixed stars (the sidereal year).

Since antiquity, astronomers have progressively refined the definition of the tropical year. The entry for "year, tropical" in the *Astronomical Almanac Online Glossary* (2015) states:

> the period of time for the ecliptic longitude of the Sun to increase 360 degrees. Since the Sun's ecliptic longitude is measured with respect to the equinox, the tropical year comprises a complete cycle of seasons, and its length is approximated in the long term by the civil (Gregorian) calendar. The mean tropical year is approximately 365 days, 5 hours, 48 minutes, 45 seconds.

An equivalent, more descriptive, definition is "The natural basis for computing passing tropical years is the mean longitude of the Sun reckoned from the precessionally moving equinox (the dynamical equinox or equinox of date). Whenever the longitude reaches a multiple of 360 degrees the mean Sun crosses the vernal equinox and a new tropical year begins" (Borkowski 1991, p. 122).

The mean tropical year in 2000 was 365.24219 ephemeris days; each ephemeris day lasting 86,400 SI seconds.[1] This is 365.24217 mean solar days (Richards 2013, p. 587).

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History

Origin

The word "tropical" comes from the Greek *tropikos* meaning "turn" (tropic 1992). Thus, the tropics of Cancer and Capricorn mark the extreme north and south latitudes where the Sun can appear directly overhead, and where it appears to "turn" in its annual seasonal motion. Because of this connection between the tropics and the seasonal cycle of the apparent position of the Sun, the word "tropical" also lent its name to the "tropical year". The early Chinese, Hindus, Greeks, and others made approximate measures of the tropical year.

Early value, precession discovery

In the 2nd century BC Hipparchus measured the time required for the Sun to travel from an equinox to the same equinox again. He reckoned the length of the year to be 1/300 of a day less than 365.25 days (365 days, 5 hours, 55 minutes, 12 seconds, or 365.24667 days). Hipparchus used this method because he was better able to detect the time of the equinoxes, compared to that of the solstices (Meeus & Savoie 1992, p. 40).

Hipparchus also discovered that the equinoctial points moved along the ecliptic (plane of the Earth's orbit, or what Hipparchus would have thought of as the plane of the Sun's orbit about the Earth) in a direction opposite that of the movement of the Sun, a phenomenon that came to be named "precession of the equinoxes". He reckoned the value as 1° per century, a value that was not improved upon until about 1000 years later, by Islamic astronomers. Since this discovery a distinction has been made between the tropical year and the sidereal year (Meeus & Savoie 1992, p. 40).

Middle Ages and the Renaissance

During the Middle Ages and Renaissance a number of progressively better tables were published that allowed computation of the positions of the Sun, Moon and planets relative to the fixed stars. An important application of these tables was the reform of the calendar.

The Alfonsine Tables, published in 1252, were based on the theories of Ptolemy and were revised and updated after the original publication; the most recent update in 1978 was by the French National Centre for Scientific Research. The length of the tropical year was given as 365 solar days 5 hours 49 minutes 16 seconds (≈ 365.24255 days). This length was used in devising the Gregorian calendar of 1582 (Meeus & Savoie 1992, p. 41).

In the 16th century Copernicus put forward a heliocentric cosmology. Erasmus Reinhold used Copernicus' theory to compute the Prutenic Tables in 1551, and gave a tropical year length of 365 solar days, 5 hours, 55 minutes, 58 seconds (365.24720 days), based on the length of a sidereal year and the presumed rate of precession. This was actually less accurate than the earlier value of the Alfonsine Tables.

Major advances in the 17th century were made by Johannes Kepler and Isaac Newton. In 1609 and 1619 Kepler published his three laws of planetary motion (McCarthy & Seidelmann 2009, p. 26). In 1627, Kepler used the observations of Tycho Brahe and Waltherus to produce the most accurate tables up to that time, the Rudolphine Tables. He evaluated the mean tropical year as 365 solar days, 5 hours, 48 minutes, 45 seconds (365.24219 days; Meeus & Savoie 1992, p. 41).

Newton's three laws of dynamics and theory of gravity were published in his *Philosophiæ Naturalis Principia Mathematica* in 1687. Newton's theoretical and mathematical advances influenced tables by Edmund Halley published in 1693 and 1749 (McCarthy & Seidelmann 2009, pp. 26-28) and provided the underpinnings of all solar system models until Albert Einstein's theory of General relativity in the 20th century.

18th and 19th century

From the time of Hipparchus and Ptolemy, the year was based on two equinoxes (or two solstices) a number of years apart, to average out both observational errors and the effects of nutation (periodic motions of the axis of rotation of the earth, the main cycle being 18.6 years) and the movement of the Sun caused by the gravitational pull of the planets. These effects did not begin to be understood until Newton's time. To model short-term variations of the time between equinoxes (and prevent them from confounding efforts to measure long-term variations) requires either precise observations or an elaborate theory of the apparent motion of the Sun. The necessary theories and mathematical tools came together in the 18th century due to the work of Pierre-Simon de Laplace, Joseph Louis Lagrange, and other specialists in celestial mechanics. They were able to express the mean longitude of the Sun as

$$L_0 = A_0 + A_1 T + A_2 T^2 \text{ days}$$

where T is the time in Julian centuries. The inverse of the derivative of L_0, dT/dL_0 gives the length of the tropical year as a linear function of T. When this is computed, an expression giving the length of the tropical year as a function of T results.

Two equations are given in the table. Both equations estimate that the tropical year gets roughly a half second shorter each century.

Tropical year coefficients

Name	Equation	Date on which T = 0
Leverrier (Meeus & Savoie 1992, p. 42)	$Y = 365.242\,196\,47 - 6.24 \times 10^{-6}\,T$	January 0.5, 1900, Ephemeris Time
Newcomb (1898)	$Y = 365.242\,198\,79 - 6.14 \times 10^{-6}\,T$	January 0, 1900, mean time

Newcomb's tables were successful enough that they were used by the joint American-British *Astronomical Almanac* for the Sun, Mercury, Venus, and Mars through 1983 (Seidelmann 1992, p. 317).

20th and 21st centuries

The length of the mean tropical year is derived from a model of the solar system, so any advance that improves the solar system model potentially improves the accuracy of the mean tropical year. Many new observing instruments became available, including

- artificial satellites
- tracking of deep space probes such as Pioneer 4 beginning in 1959 (Jet Propulsion Laboratory 2005)
- radars able to measure the distance to other planets beginning in 1961 (Butrica 1996)
- lunar laser ranging since the 1969 Apollo 11 left the first of a series of retroreflectors which allow greater accuracy than reflectorless measurements
- artificial satellites such as LAGEOS (1976) and the Global Positioning System (initial operation in 1993)
- Very Long Baseline Interferometry which finds precise directions to quasars in distant galaxies, and allows determination of the Earth's orientation with respect to these objects whose distance is so great they can be considered to show minimal space motion (McCarthy & Seidelmann 2009, p. 265).

The complexity of the model used for the solar system must be limited to the available computation facilities. In the 1920s punched card equipment came into use by L. J. Comrie in Britain. At the *American Ephemeris* an electromagnetic computer, the IBM Selective Sequence Electronic Calculator was used since 1948. When modern computers became available, it was possible to compute ephemerides using numerical integration rather than general theories; numerical integration came into use in 1984 for the joint US-UK almanacs (McCarthy & Seidelmann 2009, p. 32).

Einstein's General Theory of Relativity provided a more accurate theory, but the accuracy of theories and observations did not require the refinement provided by this theory (except for the advance of the perihelion of Mercury) until 1984. Time scales incorporated general relativity beginning in the 1970s (McCarthy & Seidelmann 2009, p. 37).

A key development in understanding the tropical year over long periods of time is the discovery that the rate of rotation of the earth, or equivalently, the length of the mean solar day, is not constant. William Ferrel in 1864 and Charles-Eugène Delaunay in 1865 indicated the rotation of the Earth was being retarded by tides. In 1921 William H Shortt invented the Shortt-Synchronome clock, the most accurate commercially produced pendulum clock; it was the first clock capable of measuring variations in the Earth's rotation. The next major time-keeping advance was the quartz clock, first built by Warren Marrison and J. W. Horton in 1927; in the late 1930s quartz clocks began to replace pendulum clocks as time standards (McCarthy & Seidelmann 2009, ch. 9).

A series of experiments beginning in the late 1930s led to the development of the first atomic clock by Louis Essen and J. V. L. Parry in 1955. Their clock was based on a transition in the cesium atom (McCarthy & Seidelmann 2009, pp. 157-9). Due to the accuracy the General Conference on Weights and Measures in 1960 redefined the second in terms of the cesium transition.[1] The atomic second, often called the SI second, was intended to agree with the ephemeris second based on Newcomb's work, which in turn makes it agree with the mean solar second of the mid-19th century (McCarthy & Seidelmann 2009, pp. 81-2, 191-7).

Time scales

As mentioned in History, advances in time-keeping have resulted in various time scales. One useful time scale is Universal Time, especially the UT1 variant, which astronomers refer to as Greenwich Mean Time in predictions supplied to the media and the general public and is the mean solar time at 0 degrees longitude (the Greenwich meridian). One second of UT is 1/86,400 of a mean solar day. This time scale is known to be somewhat variable. Since all civil calendars count actual solar days, they must ultimately be based on UT (but the actual timing of official midnight is based on UTC).

The other time scale has two parts. Ephemeris time (ET) is the independent variable in the equations of motion of the solar system, in particular, the equations in use from 1960 to 1984 (McCarthy & Seidelmann 2009, p. 378). That is, the length of the second used in the solar system calculations could be adjusted until the length that gives the best agreement with observations is found. With the introduction of atomic clocks in the 1950s, it was found that ET could be better realized as atomic time. This also means that ET is a uniform time scale, as is atomic time. ET was given a new name, Terrestrial Time (TT), and for most purposes ET = TT = International Atomic Time + 32.184 SI seconds. As of January 2017, TT is ahead of UT1 by 69.184 seconds (International Earth Rotation Service 2017; McCarthy & Seidelmann 2009, pp. 86-7).

As explained below, long-term estimates of the length of the tropical year were used in connection with the reform of the Julian calendar, which resulted in the Gregorian calendar. Participants in that reform were only partially aware of the non-uniform rotation of the Earth, but now this can be taken into account to some degree. The amount that TT is ahead of UT1 is known as ΔT, or Delta T. The table below gives Morrison and Stephenson's (S & M) 2004 estimates and standard errors (σ) for dates significant in the process of developing the Gregorian calendar.

Event	Year	Nearest S & M Year	ΔT	σ
Julian calendar begins	−44	0	2h56m20s	4m20s
First Council of Nicaea	325	300	2h8m	2m
Gregorian calendar begins	1582	1600	2m	20s
low-precision extrapolation	4000		4h13m	
low-precision extrapolation	10,000		2d11h	

The low-precision extrapolations are computed with an expression provided by Morrison & Stephenson (2004)

$$\Delta T \text{ in seconds} = -20 + 32t^2$$

where t is measured in Julian centuries from 1820. The extrapolation is provided only to show ΔT is not negligible when evaluating the calendar for long periods; Borkowski (1991, p. 126) cautions that "many researchers have attempted to fit a parabola to the measured ΔT values in order to determine the magnitude of the deceleration of the Earth's rotation. The results, when taken together, are rather discouraging."

Length of tropical year

An oversimplified definition of the tropical year would be the time required for the Sun, beginning at a chosen ecliptic longitude, to make one complete cycle of the seasons and return to the same ecliptic longitude. Before considering an example, the equinox must be examined. There are two important planes in solar system calculations, the plane of the ecliptic (the Earth's orbit around the Sun), and the plane of the celestial equator (the Earth's equator projected into space). These two planes intersect in a line. This *direction* is given the symbol ♈ (the symbol looks like the horns of a ram because it used to be toward the constellation Aries. The opposite *direction* is given the symbol ♎ (because it used to be toward Libra). Because of the precession of the equinoxes and nutation these directions change, compared to the direction of distant stars and galaxies, whose directions have no measurable motion due to their great distance (see International Celestial Reference Frame).

The ecliptic longitude of the Sun is the angle between ♈ and the Sun, measured eastward along the ecliptic. This creates a complicated measurement, because as the Sun is moving, the direction the angle is measured from is also moving. It is convenient to have a fixed (with respect to distant stars) direction to measure from; the direction of ♈ at noon January 1, 2000 fills this role and is given the symbol ♈$_0$.

Using the oversimplified definition, there was an equinox on March 20, 2009, 11:44:43.6 TT. The 2010 March equinox was March 20, 17:33:18.1 TT, which gives a duration of 365 days 5 hours 48 minutes 34.5 seconds (Astronomical Applications Dept., 2009). While the Sun moves, ♈ moves in the opposite direction . When the Sun and ♈ met at the 2010 March equinox, the Sun had moved east 359°59'09" while ♈ had moved west 51" for a total of 360° (all with respect to ♈$_0$; Seidelmann 1992, p. 104, expression for p_A).

If a different starting longitude for the Sun is chosen, the duration for the Sun to return to the same longitude will be different. This is because although ♈ changes at a nearly steady rate[2] there is considerable variation in the angular speed of the Sun. Thus, the 50 or so arcseconds that the Sun does not have to move to complete the tropical year "saves" varying amounts of time depending on the position in the orbit.

Mean time interval between equinoxes

As already mentioned, there is some choice in the length of the tropical year depending on the point of reference that one selects. But during the period when return of the Sun to a chosen longitude was the method in use by astronomers, one of the equinoxes was usually chosen because it was easier to detect when it occurred. When tropical year measurements from several successive years are compared, variations are found which are due to nutation, and to the planetary perturbations acting on the Sun. Meeus & Savoie (1992, p. 41) provided the following examples of intervals between northward equinoxes:

	days	hours	min	s
1985–1986	365	5	48	58
1986–1987	365	5	49	15
1987–1988	365	5	46	38
1988–1989	365	5	49	42
1989–1990	365	5	51	06

Until the beginning of the 19th century, the length of the tropical year was found by comparing equinox dates that were separated by many years; this approach yielded the *mean* tropical year (Meeus & Savoie 1992, p. 42).

Values of mean time intervals between equinoxes and solstices were provided by Meeus & Savoie (1992, p. 42) for the years 0 and 2000.

	Year 0	Year 2000
Between two Northward equinoxes	365.242 137 days	365.242 374 days
Between two Northern solstices	365.241 726	365.241 626
Between two Southward equinoxes	365.242 496	365.242 018
Between two Southern solstices	365.242 883	365.242 740
Mean tropical year (Laskar's expression)	365.242 310	365.242 189

Mean tropical year current value

The mean tropical year on January 1, 2000 was 365.242 189 7 or 365 ephemeris days, 5 hours, 48 minutes, 45.19 seconds. This changes slowly; an expression suitable for calculating the length of a tropical year in ephemeris days, between 8000 BC and 12000 AD is

$$365.2421896698 - 6.15359 \times 10^{-6}T - 7.29 \times 10^{-10}T^2 + 2.64 \times 10^{-10}T^3$$

where T is in Julian centuries of 36,525 days of 86,400 SI seconds measured from noon January 1, 2000 TT (in negative numbers for dates in the past; McCarthy & Seidelmann 2009, p. 18, calculated from planetary model of Laskar 1986).

Modern astronomers define the tropical year as time for the Sun's mean longitude to increase by 360°. The process for finding an expression for the length of the tropical year is to first find an expression for the Sun's mean longitude (with respect to ♈︎), such as Newcomb's expression given above, or Laskar's expression (1986, p. 64). When viewed over a one-year period, the mean longitude is very nearly a linear function of Terrestrial Time. To find the length of the tropical year, the mean longitude is differentiated, to give the angular speed of the Sun as a function of Terrestrial Time, and this angular speed is used to compute how long it would take for the Sun to move 360° (Meeus & Savoie 1992, p. 42; Astronomical Almanac for the year 2011, L8).

The above formulae give the length of the tropical year in ephemeris days (equal to 86,400 SI seconds), not solar days. It is the number of solar days in a tropical year that is important for keeping the calendar in synch with the seasons (see below).

Calendar year

The Gregorian calendar, as used for civil and scientific purposes, is an international standard. It is a solar calendar that is designed to maintain synchrony with the mean tropical year (Dobrzycki 1983, p. 123). It has a cycle of 400 years (146,097 days). Each cycle repeats the months, dates, and weekdays. The average year length is 146,097/400 = $365^{97}/_{400}$ = 365.2425 days per year, a close approximation to the mean tropical year (Seidelmann 1992, pp. 576-81).

The Gregorian calendar is a reformed version of the Julian calendar. By the time of the reform in 1582, the date of the vernal equinox had shifted about 10 days, from about March 21 at the time of the First Council of Nicaea in 325, to about March 11. According to North (1983), the real motivation for reform was not primarily a matter of getting agricultural cycles back to where they had once been in the seasonal cycle; the primary concern of Christians was the correct observance of Easter. The rules used to compute the date of Easter used a conventional date for the vernal equinox (March 21), and it was considered important to keep March 21 close to the actual equinox (North 1983, pp. 75-76).

If society in the future still attaches importance to the synchronization between the civil calendar and the seasons, another reform of the calendar will eventually be necessary. According to Blackburn and Holford-Strevens (who used Newcomb's value for the tropical year) if the tropical year remained at its 1900 value of 365.242 198 781 25 days the Gregorian calendar would be 3 days, 17 min, 33 s behind the Sun after 10,000 years. Aggravating this error, the length of the tropical year (measured in Terrestrial Time) is decreasing at a rate of approximately 0.53 s per century. Also, the mean solar day is getting longer at a rate of about 1.5 ms per century. These effects will cause the calendar to be nearly a day behind in 3200. The number of solar days in a "tropical millennium" is decreasing by about 0.06 per millennium (neglecting the oscillatory changes in the real length of the tropical year).[3] This means there should be fewer and fewer leap days as time goes on. A possible reform would be to omit the leap day in 3200, keep 3600 and 4000 as leap years, and thereafter make all centennial years common except 4500, 5000, 5500, 6000, etc. But the quantity ΔT is not sufficiently predictable to form more precise proposals (Blackburn & Holford-Strevens 2003, p. 692).

See also

- Anomalistic year
- Gregorian calendar
- Sidereal and tropical astrology

Notes

1. "The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom." CGPM (1960, Resolution 9, as quoted in Bureau International des Poids et Mesures 2006, 133)
2. The expression for p_A in Seidelmann 1992, p. 104 shows the progression per 365 days of TT increases steadily from 50.25865 arcseconds (January 1, 2009 – January 1, 2010) to 50.25889 arcseconds (December 1, 2009 – December 1, 2010).
3. 365242×1.5/8640000.

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