

# Apsidal precession

In celestial mechanics, **apsidal precession** or **orbital precession** is the precession (rotation) of the orbit of a celestial body. More precisely, it is the gradual rotation of the line joining the apsides of an orbit, which are the points of closest and farthest approach. Perihelion is the closest point to the Sun. The apsidal precession is the first derivative of the argument of periapsis, one of the six primary orbital elements of an orbit.

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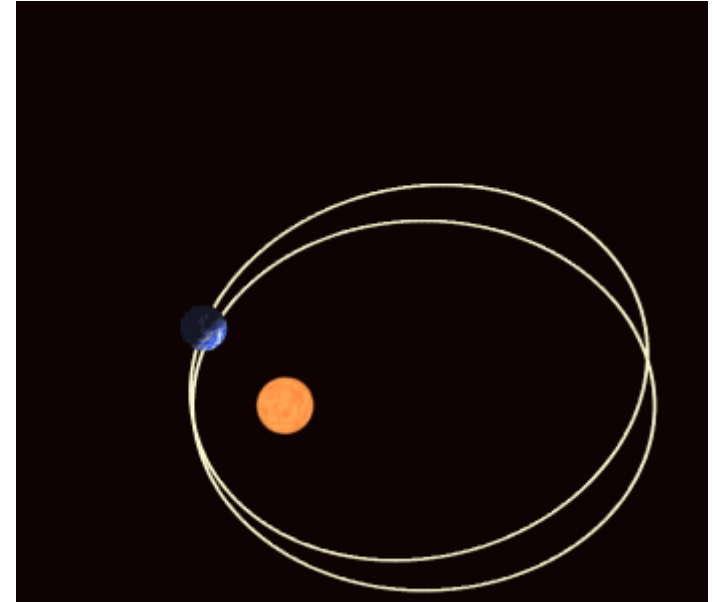
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## History

The ancient Greek astronomer Hipparchos noted the apsidal precession of the Moon's orbit;<sup>[1]</sup> it is corrected for in the Antikythera Mechanism (circa 80 BCE) with the almost exactly accurate value of 8.88 years per full cycle, correct within 0.34%.<sup>[2]</sup> The precession of the solar apsides was discovered in the eleventh century by al-Zarqālī.<sup>[3]</sup> The lunar apsidal precession was not accounted for in Claudius Ptolemy's Almagest, and as a group these precessions, the result of a plethora of phenomena, remained difficult to account for until the 20th century when the last unidentified part of Mercury's precession was precisely predicted in Albert Einstein's general theory of relativity.<sup>[4]</sup>

## Calculation



Planets orbiting the Sun follow elliptical (oval) orbits that rotate gradually over time (apsidal precession). The eccentricity of this ellipse and the precession rate of the orbit are exaggerated for visualization. Most orbits in the Solar System have a much smaller eccentricity and precess at a much slower rate, making them nearly circular and stationary.

A variety of factors can lead to periastron precession such as general relativity, stellar quadrupole moments, mutual star–planet tidal deformations, and perturbations from other planets.<sup>[5]</sup>

$$\omega_{\text{total}} = \omega_{\text{General Relativity}} + \omega_{\text{quadrupole}} + \omega_{\text{tide}} + \omega_{\text{perturbations}}$$

For Mercury, the perihelion precession rate due to general relativistic effects is 43" per century. By comparison, the precession due to perturbations from the other planets in the Solar System is 532" per century, whereas the oblateness of the Sun (quadrupole moment) causes a negligible contribution of 0.025" per century.<sup>[6][7]</sup>

From classical mechanics, if stars and planets are considered to be purely spherical masses, then they will obey a simple  $1/r^2$ , inverse-square law, relating force to distance and hence execute closed elliptical orbits. Non-spherical mass effects are caused by the application of external potential(s): the centrifugal potential of spinning bodies like the spinning of pizza dough causes flattening between the poles and the gravity of a nearby mass raises tidal bulges. Rotational and net tidal bulges create gravitational quadrupole fields ( $1/r^3$ ) that lead to orbital precession.

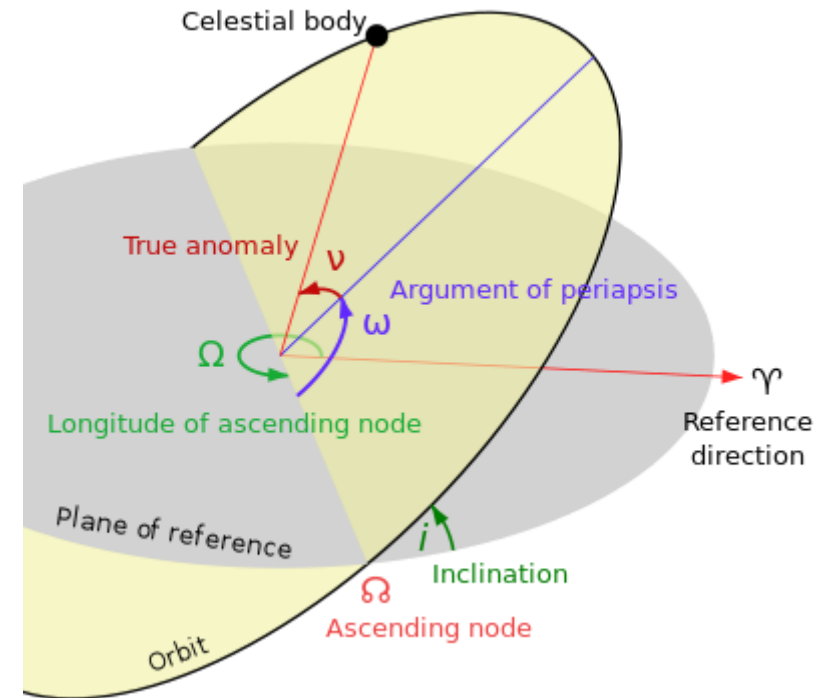
Total apsidal precession for isolated very hot Jupiters is, considering only lowest order effects, and broadly in order of importance

$$\omega_{\text{total}} = \omega_{\text{tidal perturbations}} + \omega_{\text{General Relativity}} + \omega_{\text{rotational perturbations}} + \omega_{\text{rotational}} + \omega_{\text{tidal}}$$

with planetary tidal bulge being the dominant term, exceeding the effects of general relativity and the stellar quadrupole by more than an order of magnitude. The good resulting approximation of the tidal bulge is useful for understanding the interiors of such planets. For the shortest-period planets, the planetary interior induces precession of a few degrees per year. It is up to 19.9 degrees per year for WASP-12b.<sup>[8][9]</sup>

## Newton's theorem of revolving orbits

Newton derived an early theorem which attempted to explain apsidal precession. This theorem is *historically* notable, but it was never widely used and it proposed forces which have been found not to exist, making the theorem invalid. This theorem of revolving orbits remained largely unknown and undeveloped for over three centuries until 1995.<sup>[10]</sup> Newton proposed that variations in the angular motion of a particle can be accounted for by the addition of a force that varies as the inverse cube of distance, without affecting the radial motion of a particle. Using a forerunner of the Taylor series, Newton generalized his theorem to all force laws provided



The orbital parameters. The line of apsides is shown in blue, and denoted by  $\omega$ . The apsidal precession is the rate of change of  $\omega$  through time ( $d\omega/dt$ ).

that the deviations from circular orbits are small, which is valid for most planets in the Solar System.. However, his theorem did not account for the apsidal precession of the Moon without giving up the inverse-square law of Newton's law of universal gravitation. Additionally, the rate of apsidal precession calculated via Newton's theorem of revolving orbits is not as accurate as it is for newer methods such as by perturbation theory.

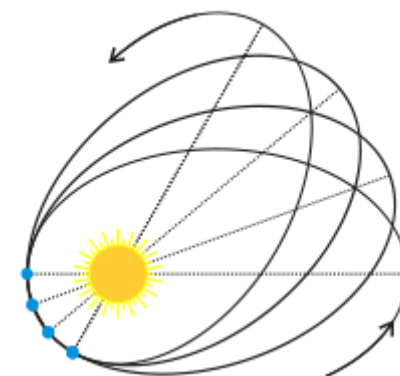
## General relativity

An apsidal precession of the planet Mercury was noted by Urbain Le Verrier in the mid-19th century and accounted for by Einstein's general theory of relativity.

Einstein showed that for a planet, the major semi-axis of its orbit being  $\alpha$ , the eccentricity of the orbit  $e$  and the period of revolution  $T$ , then the apsidal precession due to relativistic effects, during one period of revolution in radians, is

$$\epsilon = 24\pi^3 \frac{\alpha^2}{T^2 c^2 (1 - e^2)}$$

where  $c$  is the speed of light.<sup>[11]</sup> In the case of Mercury, half of the greater axis is circa  $5.79 \times 10^{10}$  m, the eccentricity of its orbit is 0.206 and the period of revolution 87.97 days or  $7.6 \times 10^6$  s. From these and the speed of light (which is  $\sim 3 \times 10^8$  m/s), it can be calculated that the apsidal precession during one period of revolution is  $\epsilon = 5.028 \times 10^{-7}$  radians ( $2.88 \times 10^{-5}$  degrees or 0.104 arcseconds). In one hundred years, Mercury makes approximately 415 revolutions around the Sun, and thus in that time, the apsidal perihelion due to relativistic effects is approximately 43 arcseconds, which corresponds almost exactly to the previously unexplained part of the measured value.



change in orbit over time

## Long-term climate

Because of apsidal precession the Earth's argument of periapsis slowly increases; it takes about 112 000 years for the ellipse to revolve once relative to the fixed stars.<sup>[12]</sup> The Earth's polar axis, and hence the solstices and equinoxes, precess with a period of about 26 000 years in relation to the fixed stars. These two forms of 'precession' combine so that it takes between 20 800 and 29 000 years (and on average 23 000 years) for the ellipse to revolve once relative to the vernal equinox, that is, for the perihelion to return to the same date (given a calendar that tracks the seasons perfectly).<sup>[13]</sup>

This interaction between the anomalistic and tropical cycle is important in the long-term climate variations on Earth, called the Milankovitch cycles. An equivalent is also known on Mars.

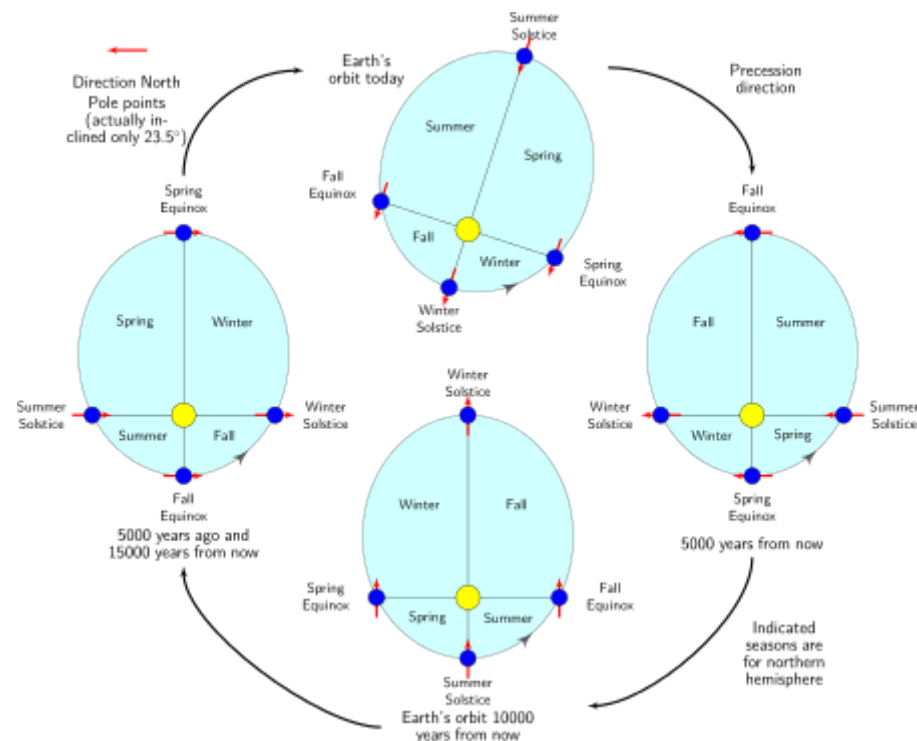
The figure illustrates the effects of precession on the northern hemisphere seasons, relative to perihelion and aphelion. Notice that the areas swept during a specific season changes through time. Orbital mechanics require that the length of the seasons be proportional to the swept areas of the seasonal quadrants, so when the orbital eccentricity is extreme, the seasons on the far side of the orbit may be substantially longer in duration.

## See also

- [Axial precession](#)
- [Nodal precession](#)
- [Hypotrochoid](#)
- [Rosetta \(orbit\)](#)
- [Spirograph](#)

## Notes

- Jones, A., Alexander (September 1991). "The Adaptation of Babylonian Methods in Greek Numerical Astronomy" ([http://www.24grammata.com/wp-content/uploads/2013/06/Jones-24grammata.com\\_.pdf](http://www.24grammata.com/wp-content/uploads/2013/06/Jones-24grammata.com_.pdf)) (PDF). *Isis*. **82** (3): 440–453. doi:10.1086/355836 (<https://doi.org/10.1086%2F355836>).
- Freeth, Tony; Bitsakis, Yanis; Moussas, Xenophon; Seiradakis, John. H.; Tselikas, A.; Mangou, H.; Zafeiropoulou, M.; Hadland, R.; et al. (30 November 2006). "Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism" ([http://www.antikythera-mechanism.gr/system/files/0608\\_Nature.pdf](http://www.antikythera-mechanism.gr/system/files/0608_Nature.pdf)) (PDF). *Nature*. **444** Supplement (7119): 587–91. Bibcode:2006Natur.444..587F (<http://adsabs.harvard.edu/abs/2006Natur.444..587F>). doi:10.1038/nature05357 (<https://doi.org/10.1038%2Fnature05357>). PMID 17136087 (<https://www.ncbi.nlm.nih.gov/pubmed/17136087>). Retrieved 20 May 2014.
- Toomer, G. J. (1969), "The Solar Theory of az-Zarqāl: A History of Errors", *Centaurus*, **14** (1): 306–36, Bibcode:1969Cent...14..306T (<http://adsabs.harvard.edu/abs/1969Cent...14..306T>), doi:10.1111/j.1600-0498.1969.tb00146.x (<https://doi.org/10.1111%2Fj.1600-0498.1969.tb00146.x>), at pp. 314-17.
- Einstein, Albert. "Explanation of the Perihelion Motion of Mercury from General Relativity Theory" (<http://www.gsjournal.net/old/eeuro/vankov.pdf>) (PDF). Retrieved August 6, 2014.
- David M. Kipping (8 August 2011). *The Transits of Extrasolar Planets with Moons* ([https://books.google.com/books?id=qXVi-zJ\\_7cC&pg=PA84](https://books.google.com/books?id=qXVi-zJ_7cC&pg=PA84)). Springer. pp. 84–. ISBN 978-3-642-22269-6. Retrieved 27 August 2013.
- Kane, S. R.; Horner, J.; von Braun, K. (2012). "Cyclic Transit Probabilities of Long-period Eccentric Planets due to Periastron Precession". *The Astrophysical Journal*. **757** (1): 105. arXiv:1208.4115 (<https://arxiv.org/abs/1208.4115>)     Bibcode:2012ApJ...757..105K (<http://adsabs.harvard.edu/abs/2012ApJ...757..105K>). doi:10.1088/0004-637x/757/1/105 (<https://doi.org/10.1088%2F0004-637x%2F757%2F1%2F105>).



Effects of apsidal precession on the seasons with the eccentricity and ap/peri-helion in the orbit exaggerated for ease of viewing. The seasons shown are in the northern hemisphere and the seasons will be reverse in the southern hemisphere at any given time during orbit. Some climatic effects follow chiefly due to the prevalence of more oceans in the Southern Hemisphere.

7. Richard Fitzpatrick (30 June 2012). *An Introduction to Celestial Mechanics* (<https://books.google.com/books?id=K5wdRGI3w8kC&pg=PA69>). Cambridge University Press. p. 69. ISBN 978-1-107-02381-9. Retrieved 26 August 2013.
8. Ragozzine, D.; Wolf, A. S. (2009). "Probing the interiors of very hot Jupiters using transit light curves". *The Astrophysical Journal*. **698** (2): 1778. arXiv:0807.2856 (<https://arxiv.org/abs/0807.2856>) . Bibcode:2009ApJ...698.1778R (<http://adsabs.harvard.edu/abs/2009ApJ...698.1778R>). doi:10.1088/0004-637x/698/2/1778 (<https://doi.org/10.1088%2F0004-637x%2F698%2F2%2F1778>).
9. Michael Perryman (26 May 2011). *The Exoplanet Handbook* (<https://books.google.com/books?id=xekY6FuKuAcC&pg=PA133>). Cambridge University Press. pp. 133–. ISBN 978-1-139-49851-7. Retrieved 26 August 2013.
10. Chandrasekhar, p. 183.
11. Hawking, Stephen. *On the Shoulders of Giants : the Great Works of Physics and Astronomy*. Philadelphia, Pennsylvania, USA: Running Press. pp. 1243, *Foundation of the General Relativity* (translated from Albert Einstein's *Die Grundlage der Allgemeine Relativitätstheorie*, first published in 1916 in *Annalen der Physik*, volume 49). ISBN 0-7624-1348-4.
12. van den Heuvel, E. P. J. (1966). "On the Precession as a Cause of Pleistocene Variations of the Atlantic Ocean Water Temperatures" (<http://gji.oxfordjournals.org/content/11/3/323.short>). *Geophysical Journal International*. **11**: 323–336. Bibcode:1966GeoJ...11..323V (<http://adsabs.harvard.edu/abs/1966GeoJ...11..323V>). doi:10.1111/j.1365-246X.1966.tb03086.x (<https://doi.org/10.1111%2Fj.1365-246X.1966.tb03086.x>).
13. *The Seasons and the Earth's Orbit* ([http://aa.usno.navy.mil/faq/docs/seasons\\_orbit.php](http://aa.usno.navy.mil/faq/docs/seasons_orbit.php)), United States Naval Observatory, retrieved 16 August 2013

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