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Angular diameter

The **angular diameter**, **angular size**, **apparent diameter**, or **apparent size** is an angular measurement describing how large a sphere or circle appears from a given point of view. In the vision sciences, it is called the visual angle, and in optics, it is the angular aperture (of a lens). The angular diameter can alternatively be thought of as the angle through which an eye or camera must rotate to look from one side of an apparent circle to the opposite side. **Angular radius** equals half the angular diameter.

Contents

Formula

The angular diameter of a circle whose plane is perpendicular to the displacement vector between the point of view and the centre of said circle can be calculated using the formula[1]

$$\delta = 2\arctan\left(\frac{d}{2D}\right),$$

in which δ is the angular diameter, and d and D are the actual diameter of and the distance to the object. When $D \gg d$, we have $\delta \approx d/D$, and the result obtained is in radians.

For a spherical object whose *actual* diameter equals d_{act}, and where D is the distance to the *centre* of the sphere, the angular diameter can be found by the formula

$$\delta = 2\arcsin\left(\frac{d_{\mathrm{act}}}{2D}\right)$$

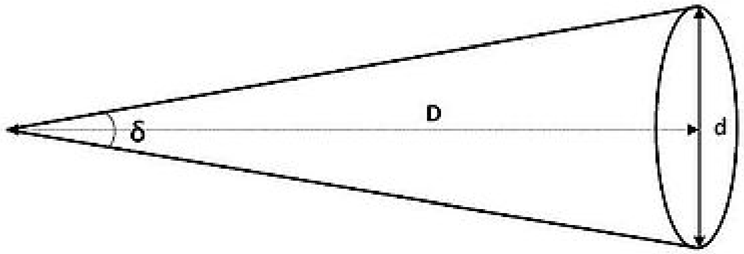


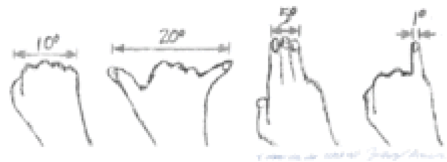
Diagram for the formula of the angular diameter

The difference is due to the fact that the apparent edges of a sphere are its tangent points, which are closer to the observer than the centre of the sphere. For practical use, the distinction is only significant for spherical objects that are relatively close, since the small-angle approximation holds for $x \ll 1$:[2]

$$\arcsin x \approx \arctan x \approx x\ .$$

Estimating angular diameter using outstretched hand

Estimates of angular diameter may be obtained by holding the hand at right angles to a fully extended arm, as shown in the figure.[3][4][5]

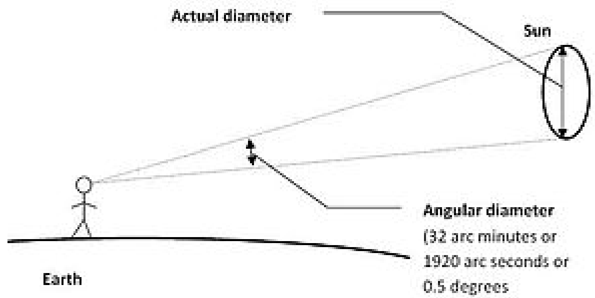


Approximate angles of 10°, 20°, 5°, and 1° for outstretched hand.

Use in astronomy

In astronomy, the sizes of celestial objects are often given in terms of their angular diameter as seen from Earth, rather than their actual sizes. Since these angular diameters are typically small, it is common to present them in arcseconds. An arcsecond is 1/3600th of one degree, and a radian is 180/π degrees, so one radian equals 3,600*180/π arcseconds, which is about 206,265 arcseconds. Therefore, the angular diameter of an object with physical diameter d at a distance D, expressed in arcseconds, is given by:[6]

$$\delta = (206{,}265)\ d\ /\ D \text{ arcseconds.}$$



Angular diameter: the angle subtended by an object

These objects have an angular diameter of 1 arcsecond ("):

- an object of diameter 1 cm at a distance of 2.06 km
- an object of diameter 725.27 km at a distance of 1 astronomical unit (AU)
- an object of diameter 45 866 916 km at 1 light-year
- an object of diameter 1 AU (149 597 871 km) at a distance of 1 parsec (pc)

Thus, the angular diameter of Earth's orbit around the Sun viewed from a distance of 1 pc is 2", as 1 AU is the mean radius of Earth's orbit.

The angular diameter of the Sun, from a distance of one light-year, is 0.03″, and that of the Earth 0.0003″. The angular diameter 0.03″ of the Sun given above is approximately the same as that of a person at a distance of the diameter of the Earth.[1] (http://www.google.com/search?hl=en&hs=3cj&q=arctan%286ft+%2F+12756.3+Km%29+in+arcseconds&btnG=Search)

This table shows the angular sizes of noteworthy celestial bodies as seen from the Earth:

Celestial body	Angular diameter or size	Relative size
Sun	31′31″ – 32′33″	30–31 times the maximum value for Venus (orange bar below) / 1891–1953″
Moon	29′20″ – 34′6″	28–32.5 times the maximum value for Venus (orange bar below) / 1760–2046″
Helix Nebula	about 16′ by 28′	
Spire in Eagle Nebula	4′40″	length is 280″
Venus	9.67″ – 1′3″	
Jupiter	29.80″ – 50.59″	
Saturn	14.50″ – 21.37″	
Mars	3.49″ – 25.13″	
Mercury	4.54″ – 13.02″	
Uranus	3.31″ – 4.08″	
Neptune	2.17″ – 2.37″	
Ceres	0.33″ – 0.84″	
Vesta	0.20″ – 0.64″	
Pluto	0.065″ – 0.116″	
"Planet Jehoshaphat" (hypothetical)[7]	0.015″ – 0.18″	

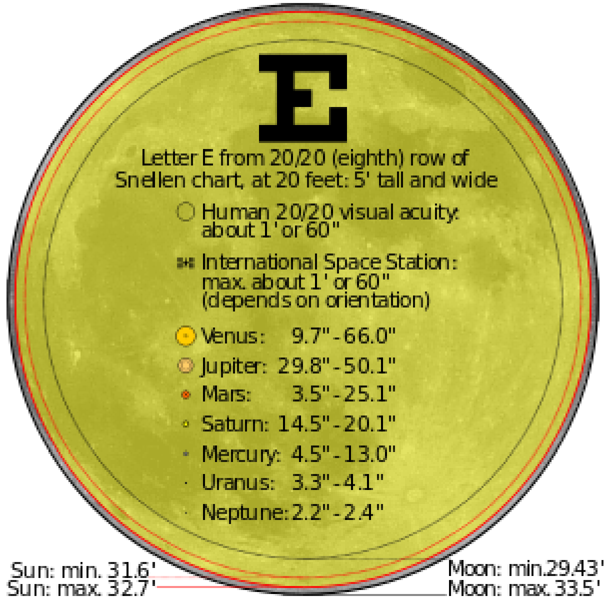
R Doradus	0.052″ – 0.062″	
Betelgeuse	0.049″ – 0.060″	
Eris	0.034″ – 0.089″	
Alphard	0.00909″	
Alpha Centauri A	0.007″	
Canopus	0.006″	
Sirius	0.005936″	
Altair	0.003″	
Deneb	0.002″	
Proxima Centauri	0.001″	
Alnitak	0.0005″	
A star like Alnitak at a distance where the Hubble space telescope would just be able to see it[8]	6×10^{-10} arcsec	

The table shows that the angular diameter of Sun, when seen from Earth is approximately 32 arcminutes (1920 arcseconds or 0.53 degrees), as illustrated above.

Thus the angular diameter of the Sun is about 250,000 times that of Sirius (Sirius has twice the diameter and its distance is 500,000 times as much; the Sun is 10^{10} times as bright, corresponding to an angular diameter ratio of 10^5, so Sirius is roughly 6 times as bright per unit solid angle).

The angular diameter of the Sun is also about 250,000 times that of Alpha Centauri A (it has about the same diameter and the distance is 250,000 times as much; the Sun is 4×10^{10} times as bright, corresponding to an angular diameter ratio of 200,000, so Alpha Centauri A is a little brighter per unit solid angle).

The angular diameter of the Sun is about the same as that of the Moon (the Sun's diameter is 400 times as large and its distance also; the Sun is 200,000 to 500,000 times as bright as the full Moon (figures vary), corresponding to an angular diameter ratio of 450 to 700, so a celestial body with a diameter of 2.5–4″ and the same brightness per unit solid angle would have the same brightness as the full Moon).



Comparison of angular diameter of the Sun, Moon and planets. To get a true representation of the sizes, view the image at a distance of 103 times the width of the "Moon: max." circle. For example, if this circle is 5 cm wide on your monitor, view it from 5.15 m away.

Even though Pluto is physically larger than Ceres, when viewed from Earth (e.g., through the Hubble Space Telescope) Ceres has a much larger apparent size.

While angular sizes measured in degrees are useful for larger patches of sky (in the constellation of Orion, for example, the three stars of the belt cover about 4.5 degrees of angular size), we need much finer units when talking about the angular size of galaxies, nebulae or other objects of the night sky. Degrees, therefore, are subdivided as follows:

- 360 degrees (°) in a full circle
- 60 arc-minutes (′) in one degree
- 60 arc-seconds (″) in one arc-minute

To put this in perspective, the full moon viewed from Earth is about $^1/_2$ degree, or 30 arc minutes (or 1800 arc-seconds). The Moon's motion across the sky can be measured in angular size: approximately 15 degrees every hour, or 15 arc-seconds per second. A one-mile-long line painted on the face of the Moon would appear to us to be about one arc-second in length.

In astronomy, it is typically difficult to directly measure the distance to an object. But the object may have a known physical size (perhaps it is similar to a closer object with known distance) and a measurable angular diameter. In that case, the angular diameter formula can be inverted to yield the Angular diameter distance to distant objects as

$$d \equiv 2D\tan\left(\frac{\delta}{2}\right).$$

In non-Euclidean space, such as our expanding universe, the angular diameter distance is only one of several definitions of distance, so that there can be different "distances" to the same object. See Distance measures (cosmology).

Non-circular objects

Many deep sky objects such as galaxies and nebulas appear as non-circular, and are thus typically given two measures of diameter: *Major Diameter* and *Minor Diameter*. For example, the Small Magellanic Cloud has a visual apparent diameter of 5° 20′ × 3° 5′.

Defect of illumination

Defect of illumination is the maximum angular width of the unilluminated part of a celestial body seen by a given observer. For example, if an object is 40 seconds of arc across and is 75 percent illuminated, the defect of illumination is 10 seconds of arc.

This photo compares the apparent sizes of Jupiter and its four Galilean moons (Callisto at maximum elongation) with the diameter of the full moon's disk during their conjunction on 10 April 2017.

See also

- Angular diameter distance
- Angular resolution
- Solid angle
- Visual acuity
- Visual angle
- Visual Angle Illusion
- List of stars with resolved images

References

1. This can be derived using the formula for the length of a cord found at http://mathworld.wolfram.com/CircularSegment.html
2. http://www.mathstat.concordia.ca/faculty/rhall/mc/arctan.pdf
3. "Archived copy" (https://web.archive.org/web/20150121044615/https://dept.astro.lsa.umich.edu/ugactivities/Labs/coords/index.html). Archived from the original (https://dept.astro.lsa.umich.edu/ugactivities/Labs/coords/index.html) on 2015-01-21. Retrieved 2015-01-21.
4. "Photographing Satellites" (https://www.bartbusschots.ie/s/2013/06/08/photographing-satellites/). 8 June 2013.
5. Wikiversity: Physics and Astronomy Labs/Angular size
6. Michael A. Seeds; Dana E. Backman (2010). *Stars and Galaxies* (7 ed.). Brooks Cole. p. 39. ISBN 978-0-538-73317-5.
7. The figures 0.015″ and 0.18″ are based on lower and upper estimates of its diameter (13 000 km and 26 000 km) at estimated aphelion and perihelion (1200 AU and 200 AU) respectively.
8. 800 000 times smaller angular diameter than that of Alnitak as seen from Earth. Alnitak is a blue star so it gives off a lot of light for its size. If it were 800 000 times further away then it would be magnitude 31.5, at the limit of what Hubble can see.

External links

- Small-Angle Formula (https://web.archive.org/web/19971007100829/http://ceres.hsc.edu/homepages/classes/astronomy/fall97/Mathematics/sec9.html)
- Visual Aid to the Apparent Size of the Planets (http://www.astronomynotes.com/solarsys/s2.htm)